Unpaid Work, Growth, and the Optimal Tax Scheme

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1. Introduction

Our society is composed of not only activities in the market economy but also activities outside the market economy. We supply labor in the labor market, and in return, require the wage income. In contrast, we devote labor to family members or others, and yet do not require the wage income. Thus, work force in our society can be divided into two classes, namely, paid work force and unpaid work force. According to the Economic Planning Agency of Japan, the globally admissible definition of unpaid work is as follows: the producer and the consumer of goods and services involving unpaid work are not necessarily the same, and self-sufficient goods and services involving unpaid work must be marketable. Housework, volunteer activities, nursing, child-rearing, and social work are considered to be good examples of goods and services involving unpaid work.

Lately, various economists mention that unpaid work occupies weighty hours of human life, and therefore, we should not neglect such activities. An empirical approach to grasp the essence of unpaid work progresses gradually. Many national bureaus of statistics in Western countries are searching for an adequate method to evaluate unpaid work. Murtgatroy and Neuberger (1995) estimated a monetary value of unpaid work in the UK. According to Murtgatroy and Neuberger, unpaid work valued as paid based on wages and salaries was about 6464 million pounds or 106% of GDP in the UK. Their calculation method is denoted by

$$\frac{\text{unpaid work valued as paid}}{\text{based on wages and salaries}}=\frac{\text{unpaid work hours}}{\text{paid work hours}}\times\text{aggregate wage in the UK.}$$

The Economic Planning Agency of Japan also estimated the monetary value of unpaid work in 1981, 1986, 1991, and 1996. It developed three calculation methods, namely, the opportunity cost method, the replacement cost method-specialist approach, and the replacement cost method-generalist approach. These calculation methods are somehow different from the Murtgatroy and Neuberger's one. However, if we recalculate the monetary value of unpaid work in Japan according to the Murtgatroy and Neuberger's method, then we obtain 239 trillion and 4658 million yen or 62% of GDP in 1996. Readers who like to know more about this can easily obtain the online report from http://www.epa.go.jp/98/g/19981105g-unpaid.html.

However, the theoretical approach in terms of unpaid work is outdistanced relative to the empirical one. To the best of my knowledge, the theoretical construction which grasps the essence of unpaid work has not yet progressed in preceding literatures. Then, the key question to be address is to the extent which we are able to map unpaid work into our economic models. Close observation reveals that as we do volunteer activities, go grocery shopping, or rear children, we tend to utilize public consumption goods such as roads, parks, public transport facilities, and so on. Therefore, in this paper, we assume that self-sufficient goods and services are composed of both unpaid work and public consumption goods.

The structure of the paper is as follows. Section 2 presents our model and section 3 solves for the competitive equilibrium in the decentralized economy. We denote the definition of the steady growth equilibrium and its analysis in section 4. In section 5, we focus on the steady growth equilibrium in the centralized economy, thereby searching for the optimal tax scheme. Section 6 provides conclusions and further insights.

2. The Model

2.1 Identical Agents

Consider identical agents who consume private consumption goods, enjoy self-sufficient goods and services involving unpaid work, and rent physical capital and human capital to firms in the competitive private consumption good market. We assume no population growth so that we can ignore the scale effect of the economy. We also assume the following intertemporal isoelastic utility function:

$$U=\int_0^\infty \frac{[C(t)(L(t))^\eta]^{1-\theta}-1}{1-\theta}e^{-\rho t}dt,\ \theta\neq 1,\ \eta>0,\ \rho>0. \tag{1}$$

where η measures the impact of self-sufficient goods and services on the welfare, U, of agents, ρ is the rate of time preference, and θ is the inverse of the intertemporal elasticity of substitution. Private consumption goods and self-sufficient goods and services at time t are denoted, respectively, by $C(t)$ and $L(t)$. The intertemporal isoelastic utility function reduces to $U=\ln C(t)+\eta\ln L(t)$ when $\theta=1$.[1)]

In our model, agents accumulate human capital in the education sector such as school or on-the-job training in firms. The following accumulation technology is considered:

$$\dot{H}(t)=B[(1-v(t))K(t)]^\beta[z(t)H(t)]^{1-\beta} \tag{2}$$

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1) Let $M=(CL^\eta)^{1-\theta}-1$ and $N=1-\theta$. Then $\ln(M+1)=(1-\theta)\ln CL^\eta$ or $M=\exp[(1-\theta)\ln CL^\eta]-1$. A partial derivative of M with respect to θ gives us $\partial M/\partial\theta=-(\ln C+\eta\ln L)\exp[(1-\theta)\ln CL^\eta]$. We also take a partial derivative of N with respect to θ in order to obtaim $\partial N/\partial\theta=-1$. Thus, by L'Hopital's rule, $\lim_{\theta\to 1}\frac{CL^\eta-1}{1-\theta}=\lim_{\theta\to 1}\frac{-(\ln C+\eta\ln L)\exp[(1-\theta)\ln CL^\eta]}{-1}=\ln C+\eta\ln L.$

where $0<\beta<1$, and $(1-v(t))$ and $z(t)$ are the fraction of physical capital and that of human capital devoted to the education sector, respectively. Thus, as in Rebelo (1991), human capital is produced by agents with a CRS technology. For simplicity, the human capital depreciation is ignored. We assume that physical capital employed in the education sector is not remunerated because the education sector is non-market.

On the other hand, the economy-wide resource constraint is denoted by

$$\dot{K}(t)=Y(t)-C(t)-G(t) \tag{3}$$

where $Y(t)$ is output at time t and $G(t)$ is government expenditure at time t. Equation (3) asserts that the rate of physical capital accumulation is the excess of current production over private consumption and government expenditure.

2.2 Firms

Firms rent physical capital at a rate, $r_K(t)$ and hire human capital at a rate, $w(t)$ from agents in each moment in time. They use physical capital and human capital to produce private consumption goods with the Cobb-Douglas technology given by

$$Y(t)=A[v(t)K(t)]^{\alpha}[u(t)H(t)]^{1-\alpha} \tag{4}$$

where $0<\alpha<1$ and $v(t)$ are $u(t)$ the fraction of physical capital and that of human capital devoted to the production of private consumption goods, respectively. Then the profit maximization problem of firms in each moment in time is denoted by

$$Max\ \pi\ where\ \pi=A(vK)^{\alpha}(uH)^{1-\alpha}-r_K vK-wuH.$$

The price of output is fixed at unity and the time subscripts are omitted here. Then the first order conditions for the profit maximizing firms suggest the physical capital rental rate and the human capital wage rate in the competitive market, respectively, by

$$r_K=\alpha A\left(\frac{vK}{uH}\right)^{\alpha-1} \text{ [2]} \tag{5}$$

and

$$w=(1-\alpha)A\left(\frac{vK}{uH}\right)^{\alpha}. \text{ [3]} \tag{6}$$

2.3 The Government

The government provides government consumption goods to agents. In order to finance government expenditure for government consumption goods, the government collects taxes from agents. The government also issue bonds. At any instant of time it makes expenditure deci-

2) We rewirite π as $\pi=uH\left[A\left(\frac{vK}{uH}\right)^{\alpha}-r_K\left(\frac{vK}{uH}\right)-w\right]$. Then, $\frac{\partial\pi}{\partial vK}=0$ implies $r_K=A\alpha\left(\frac{vK}{uH}\right)^{\alpha-1}$.

3) Since, in full market equilibrium, w will be determined such that the profit of firms is zero. Thus we find that equation (6) holds for any size of uH. See Barro (1991) and Barro and Sala-i-Martin (1992), (1995).

sions, taxation decisions, and financing decisions. Thus, the government budget constraint is expressed as

$$\dot{S}(t)=r(t)S(t)+G(t)-T(t). \tag{7}$$

where $S(t)$ are government bonds, $T(t)$ are tax receipts, and $r(t)$ is a real interest rate on government bonds. This equation asserts that the government deficit, which consists of government expenditure plus interest payments on its outstanding debt, less tax receipts, must be financed by issuing additional debt. For convenience, we assume that all of the government expenditure is financed by tax receipts, thereby implying that $G(t)=T(t)$ holds in each moment.[4)]

2.4 Self-sufficient goods and services involving unpaid work

How we can map self-sufficient goods and services involving unpaid work such as volunteer activities into our model is one of the main questions to be address in this paper. In our model, we assume that self-sufficient goods and services is composed of both human capital and public consumption goods. Thus, human capital which is devoted to neither the output production sector nor the education sector is used as one of the inputs for the production of self-sufficient goods and services. Further, we assume that the production technology of self-sufficient goods and services also takes the Cobb-Douglas form:

$$L(t)=[(1-u(t)-z(t))H(t)]^{\phi}G(t)^{1-\phi}. \tag{8}$$

where $0<\phi<1$.

3. A Decentralized Economy

In this section we analyze a decentralized economy of our model. First of all, Equation (8) allows us to rewrite the intertemporal isoelastic utility function of agents given by equation (1) as follows:

$$U=\int_0^{\infty}\frac{\left[C(t)[(1-u(t)-z(t))H(t)]^{\phi\eta}G(t)^{(1-\phi)\eta}\right]^{1-\theta}-1}{1-\theta}e^{-\rho t}dt. \tag{9}$$

Further, we assume that the government sets the level of public consumption goods being proportional to that of private consumption goods and agents know this government policy. That is, the government sets $G(t)$ in accordance with the rule $G(t)/C(t)=(1-\phi)\eta$. This assumption can be supported empirically. Figure 1 gives the $G-C$ ratio in Germany, France, the US, and Japan from 1972 to 1992. In Figure 1 we see that the $G-C$ ratio of four representative countries does not fluctuate remarkably.

The justification of this assumption can also be supported theoretically and will be discussed in

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4) See Barro (1990), (1991).

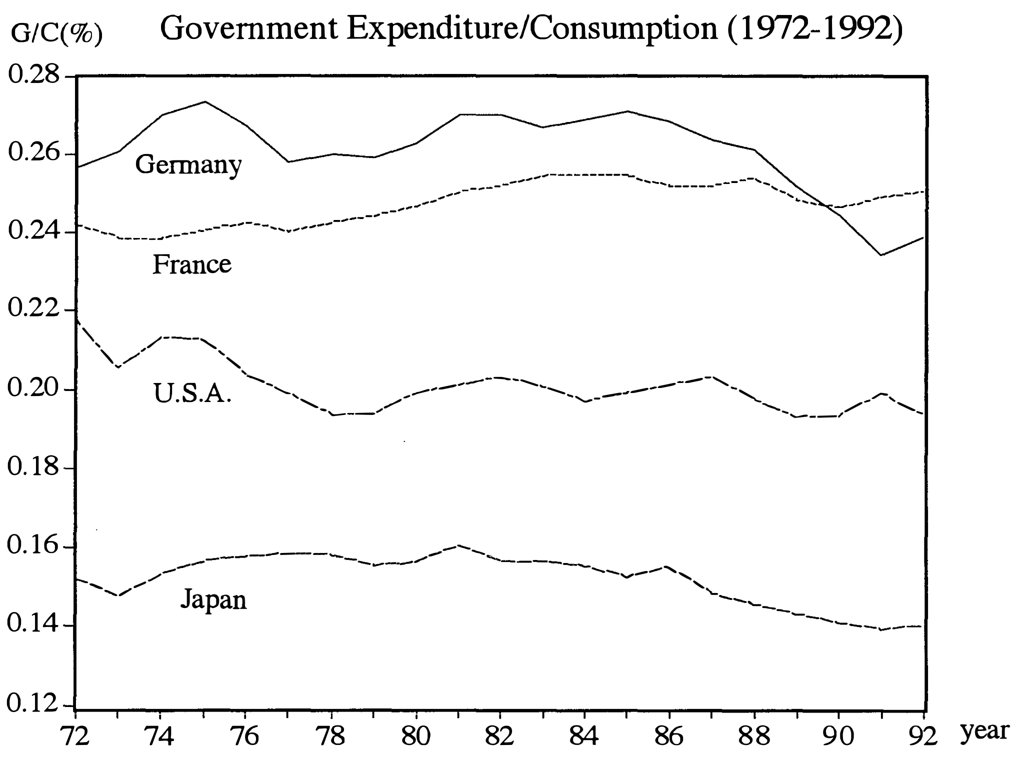
the later section concerning a centralized economy of our model.

On one hand, we can construct the budget constraint of the typical agent by equations (3) and (7) and by taking into account that $G=T=\tau_K r_K vK+\tau_H wuH+\tau_C C$ where $0\leq\tau_K<1$, $0\leq\tau_H<1$ and $0\leq\tau_C<1$ are the capital income tax, the wage income tax and the consumption tax, respectively. Then we may denote his budget constraint by

$$\dot{K}(t)+\dot{S}(t)=(1-\tau_K(t))r_K(t)vK(t)+(1-\tau_H(t))w(t)uH(t)+r(t)S(t)-(1+\tau_C(t))C(t). \quad (10)$$

We ignore the physical capital depreciation as we do the human capital depreciation. Equation (10) states that agent purchases private consumption goods and pays taxes out of the income generated by his holdings of government bonds and those of physical capital and human capital devoted to the production of output. Then, remembering $G(t)/C(t)=(1-\phi)\eta$, he chooses paths of C, S, K, H, u, v, and z to maximize equation (9) subject to equations (2) and (10), the initial physical capital stock, the initial human capital stock, and initial government bond holdings taking paths of r_K, w, r, τ_K, τ_H, and τ_C as given. Therefore, we set up the current value Hamiltonian,

Figure 1



Source: Penn World Tables, Mark 5.6

$$J(C, K, H, S, u, v, z, \lambda_1, \lambda_2, t)=\frac{\Big[[(1-\phi)\eta]^{(1-\phi)\eta}[(1-u-z)H]^{\phi\eta}C^{(1-\phi)\eta+1}\Big]^{1-\theta}-1}{1-\theta}$$

$$+\lambda_1\Big[(1-\tau_K(t))r_K(t)vK+(1-\tau_H(t))w(t)uH+r(t)S-(1+\tau_C(t))C\Big]$$

$$+\lambda_2\Big[B[(1-v)K]^{\beta}[zH]^{1-\beta}\Big]$$

where λ_1 and λ_2 are the shadow value of physical capital (or government bonds) and that of human capital, respectively. Then the necessary conditions derived from "Maximum Principle" should involve,

$$\frac{\partial J}{\partial C}=0;\ [(1-\phi)\eta+1][(1-\phi)\eta]^{(1-\phi)\eta}C^{(1-\phi)\eta}[(1-u-z)H]^{\phi\eta}[D]^{-\theta}-(1+\tau_C)\lambda_1=0: \tag{11}$$

$$\frac{\partial J}{\partial u}=0;\ -\phi\eta[(1-\phi)\eta]^{(1-\phi)\eta}C^{(1-\phi)\eta+1}[(1-u-z)H]^{\phi\eta-1}[D]^{-\theta}+(1-\tau_H)w\lambda_1=0: \tag{12}$$

$$\frac{\partial J}{\partial z}=0;\ -\phi\eta[(1-\phi)\eta]^{(1-\phi)\eta}C^{(1-\phi)\eta+1}[(1-u-z)H]^{\phi\eta-1}[D]^{-\theta} \tag{13}$$

$$+(1-\beta)B\Big[\frac{(1-v)K}{zH}\Big]^{\beta}\lambda_2=0:$$

$$\frac{\partial J}{\partial v}=0;\ (1-\tau_K)r_K\lambda_1-B\beta\Big[\frac{(1-v)K}{zH}\Big]^{\beta-1}\lambda_2=0: \tag{14}$$

$$-\dot{\lambda}_1+\lambda_1\rho=\frac{\partial J}{\partial K};\ -\dot{\lambda}_1+\lambda_1\rho=(1-\tau_K)r_Kv\lambda_1+B\beta(1-v)\Big[\frac{(1-v)K}{zH}\Big]^{\beta-1}\lambda_2: \tag{15}$$

$$-\dot{\lambda}_1+\lambda_1\rho=\frac{\partial J}{\partial S};\ -\dot{\lambda}_1+\lambda_1\rho=r\lambda_1: \tag{16}$$

$$-\dot{\lambda}_2+\lambda_2\rho=\frac{\partial J}{\partial H};$$

$$-\dot{\lambda}_2+\lambda_2\rho=\phi\eta(1-u-z)[(1-\phi)\eta]^{(1-\phi)\eta}C^{(1-\phi)\eta+1}[(1-u-z)H]^{\phi\eta-1}[D]^{-\theta}$$

$$+(1-\tau_H)wu\lambda_1+(1-\beta)Bz\Big[\frac{(1-v)K}{zH}\Big]^{\beta}\lambda_2 \tag{17}$$

where $D=[(1-\phi)\eta]^{(1-\phi)\eta}C^{(1-\phi)\eta+1}[(1-u-z)H]^{\phi\eta}$, and transversality conditions,

$$\lim_{t\to\infty}\lambda_1(t)K(t)e^{-\rho t}=0;$$

$$\lim_{t\to\infty}\lambda_1(t)S(t)e^{-\rho t}=0;$$

$$\lim_{t\to\infty}\lambda_2(t)H(t)e^{-\rho t}=0.$$

Then, combining equations (12) and (13) yields the condition,

$$(1-\tau_H)w\lambda_1=(1-\beta)B\Big[\frac{(1-v)K}{zH}\Big]^{\beta}\lambda_2. \tag{18}$$

After multiplying both sides of equation (14) by $(1-v)$, we substitute the resultant equation into equation (15) in order to obtain

$$-\dot{\lambda}_1+\lambda_1\rho=(1-\tau_K)r_K\lambda_1=r\lambda_1. \tag{19}$$

Likewise, after multiplying both sides of equation (13) by $(1-u-z)$, we substitute the resultant equation into equation (17) in order to obtain

$$-\dot{\lambda}_2+\lambda_2\rho=(1-\tau_H)wu\lambda_1+(1-\beta)B(1-u)\left[\frac{(1-v)K}{zH}\right]^{\beta}\lambda_2. \tag{20}$$

Further, after multiplying both sides of equation (28) by u, we substitute the resultant equation into equation (20) in order to obtain the condition,

$$-\dot{\lambda}_2+\lambda_2\rho=(1-\beta)B\left[\frac{(1-v)K}{zH}\right]^{\beta}\lambda_2. \tag{21}$$

Then, equations (14) and (18) give the condition which tells us that the after tax human capital return-physical capital return ratio is proportional to the physical capital-human capital ratio, or

$$\frac{(1-\tau_H)w}{(1-\tau_K)r_K}=\frac{(1-\beta)(1-v)K}{\beta zH}.$$

Equations (5) and (6) with the above ratio give us the condition,

$$\frac{v}{u}=\frac{\alpha(1-\beta)(1-\tau_K)(1-v)}{\beta(1-\alpha)(1-\tau_H)z}. \tag{22}$$

4. The Steady Growth Equilibrium

We now ready to denote the definition of the steady growth equilibrium in our model. The steady growth equilibrium is the state where economic variables (C, K, H, S, G, L, and Y) grow at constant rates, while factor allocations (u, z, and v) remain constant over time or u, z, and v do not grow. In remaining this paper, we assume the existence of the steady growth equilibrium and focus on such equilibrium. Then let growth rates of economic variables in the steady growth equilibrium, say, equilibrium steady growth rates, be $\dot{C}/C=\psi_C$, $\dot{H}/H=\psi_H$, $\dot{S}/S=\psi_S$, $\dot{G}/G=\psi_G$, $\dot{L}/L=\psi_L$, and $\dot{Y}/Y=\psi_Y$. Keeping these expressions in mind, we take time derivatives of both sides of equation (11) in order to obtain

$$[(1-\theta)(1-\phi)\eta-\theta]\psi_C+(1-\theta)\phi\eta\psi_H=\frac{\dot{\lambda}_1}{\lambda_1}. \tag{23}$$

Combining equations (5), (29), and (23) yields

$$[\theta-(1-\theta)(1-\phi)\eta]\psi_C-(1-\theta)\phi\eta\psi_H+\rho=(1-\tau_K)\alpha A\left(\frac{vK}{uH}\right)^{\alpha-1}. \tag{24}$$

By taking time derivatives of both sides of equation (24), we discover the condition, $\psi_C=\psi_H$, thereby asserting that the physical capital-human capital ratio K/H, must be constant in the steady growth equilibrium.

On the other hand, equations (3), (7), (10), and conditions, $G=T=\tau_K r_K vK+\tau_H wuH+\tau_C C$ and $G(t)/C(t)=(1-\phi)\eta$ imply that

$$\psi_K=Av\left(\frac{vK}{uH}\right)^{\alpha-1}-[(1-\phi)\eta+1]\frac{C}{K}. \tag{25}$$

As we already know that K/H is constant in the steady growth equilibrium, equation (25) implies that C/K must also be constant. Thus, $\psi_C=\psi_K=\psi_H=\psi$ holds or equilibrium growth rates of consumption, physical capital, and human capital have to share the same growth rate. Further, the specification of self-sufficient goods and services involving unpaid work, the production function of output, and the constant consumption-government expenditure ratio over time imply that equilibrium steady growth rates of output, public consumption goods, and self-sufficient goods and services involving unpaid work also share the common growth rate, ψ. To sum up, we have $\psi_C=\psi_K=\psi_H=\psi_G=\psi_Y=\psi_L=\psi$.

Taking time derivatives of both sides of equation (18) or equation (14) implies

$$\frac{\dot{\lambda}_1}{\lambda_1}=\frac{\dot{\lambda}_2}{\lambda_2}. \tag{26}$$

Thus, equations (5), (19), (21), and (26) reveal that

$$r=(1-\tau_K)\alpha A\left(\frac{vK}{uH}\right)^{\alpha-1}=(1-\beta)B\left[\frac{(1-v)K}{zH}\right]^{\beta}. \tag{27}$$

We combine equations (22) and (27) in order to obtain

$$\left(\frac{vK}{uH}\right)^{\alpha-1}=\left(\frac{(1-\beta)B}{(1-\tau_K)\alpha A}\right)^{\frac{1-\alpha}{1-\alpha+\beta}}\left(\frac{\beta(1-\alpha)(1-\tau_H)}{\alpha(1-\beta)(1-\tau_K)}\right)^{\frac{(1-\alpha)\beta}{1-\alpha+\beta}} \tag{28}$$

or, alternatively,

$$\left(\frac{(1-v)K}{zH}\right)^{\beta}=\left(\frac{(1-\tau_K)\alpha A}{(1-\beta)B}\right)^{\frac{\beta}{1-\alpha+\beta}}\left(\frac{\beta(1-\alpha)(1-\tau_H)}{\alpha(1-\beta)(1-\tau_K)}\right)^{\frac{(1-\alpha)\beta}{1-\alpha+\beta}}. \tag{29}$$

Either equation (28) or equation (29) can be inserted into equation (27) in order to possess the rate of return on bond holdings in the steady growth equilibrium.

$$r=\left[(1-\tau_K)^{\alpha\beta}(1-\tau_H)^{(1-\alpha)\beta}(A\alpha)^{\beta}[(1-\beta)B]^{1-\alpha}\left[\frac{\beta(1-\alpha)}{\alpha(1-\beta)}\right]^{(1-\alpha)\beta}\right]^{\frac{1}{1-\alpha+\beta}}. \tag{30}$$

Since the condition, $\psi_C=\psi_K=\psi_H=\psi_G=\psi_Y=\psi_L=\psi$ allows us to rewrite equation (23) as

$$-[\theta-(1-\theta)\eta]\psi=\frac{\dot{\lambda}_1}{\lambda_1}, \tag{31}$$

by combining equations (19), (30), and (31), we may specify the equilibrium steady growth rate as

$$\psi=\frac{\left[(1-\tau_K)^{\alpha\beta}(1-\tau_H)^{(1-\alpha)\beta}(A\alpha)^{\beta}[(1-\beta)B]^{1-\alpha}\left[\frac{\beta(1-\alpha)}{\alpha(1-\beta)}\right]^{(1-\alpha)\beta}\right]^{\frac{1}{1-\alpha+\beta}}-\rho}{\theta-(1-\theta)\eta}. \tag{32}$$

Then we may summarize how each tax affects the long-run growth of the decentralized economy in proposition 1.

Proposition 1

When the production of self-sufficient goods and services is expressed as equation (8), an increase in the rate of tax on physical capital or that on human capital deteriorates the equilibrium steady growth rate of the decentralized economy, while an increase in the rate of tax on consump-

tion has no growth effect.

Proposition 1 is consistent with the claim by Rebelo (1991) and Stocky and Rebelo (1995) that a change in the consumption tax has no growth effect and is inconsistent with the claim by Turnovsky (1995) that an increase in the consumption tax has a negative growth effect. Rebelo considers a case in which the additional tax receipts from an increase in the consumption tax is not rebated in a lump-sum fashion to consumers, and therefore, the consumption tax has no growth effect. On the other hand, Turnovsky considers a case in which the utility enhancing government expenditure is in the form of consumption goods that interact with private consumption goods in the utility function of a representative agent. In his model the utility enhancing government expenditure is linked to output whose production function exhibits AK technology. An important feature of his analysis is that the rate of return on capital has two components: the after-tax rate of return and a utility return resulting from the fact that an increase in the capital stock rises the consumption benefits to the representative agent from government expenditure. This utility return is the main reason that the consumption tax has a growth effect.

Equation (32) implies that the capital income tax and the wage income tax have no growth effect if $\beta=0$. When $\beta=0$, our model becomes the model developed by Lucas (1988) with goods and service involving unpaid work. In this case, the rate of return on bond holdings in the steady growth equilibrium is B, and therefore, the capital income tax and the wage income tax have no growth effect. Thus, how we assume the accumulation technology of human capital is one of the important factors to examine the growth effect of each tax. Alternatively, if we assume that public consumption goods is nothing but self-sufficient goods and services or $L(t)=G(t)$, then the entire human capital is denoted to the production sector and the education sector. In this case, we go back to the conclusion that an increase in the consumption tax has no growth effect.

In contrast, if we assume that $L(t)=[1-u(t)-z(t)]^{\phi}G(t)^{1-\phi}$ or self-sufficient goods and services require one unit of human capital and public consumption goods, the consumption tax has a growth effect. This is because the term, $u+z$ enters into the expression of the rate of return on bond holdings in the steady growth equilibrium and the consumption tax has a level effect on $u+z$.[5)]

Next, we analyze how changes in the three tax rates can affect the consumer's decision of factor allocations. Dividing equation (12) by H, the equilibrium steady growth rate can also be denoted by

$$\psi=B\left(\frac{(1-v)K}{zH}\right)^{\beta}z. \tag{33}$$

5) In this case, equation (30) becomes $r=\left[(1-\tau_K)^{\alpha\beta}(1-\tau_H)^{(1-\alpha)\beta}(A\alpha)^{\beta}[(1-\beta)B]^{1-\alpha}\left[\frac{\beta(1-\alpha)}{\alpha(1-\beta)}\right]^{(1-\alpha)\beta}(u+z)^{1-\alpha}\right]^{\frac{1}{1-\alpha+\beta}}$.

Thus, combining equations (27), (32), and (33) yields human capital allocation in the education sector in the steady growth equilibrium,

$$\hat{z}=\frac{(1-\beta)\psi}{r}\text{[6]} \tag{34}$$

where $r=\rho+[\theta-(1-\theta)\eta]\psi$. Note that we have $r>(1-\beta)\psi$ since $0<\hat{z}<1$ holds. Human capital allocation in the output production sector in the steady growth equilibrium can be derived by combining equations (3), (22), (27), and (32). Then we have

$$\hat{u}=\frac{(1-\alpha)(1-\tau_H)\Big[r(r-(1-\beta)\psi)[(1-\phi)\eta+1]^2+(1+\tau_C)\phi\eta\beta\psi^2\Big]}{r(1+\tau_C)\phi\eta[r-(1-\tau_K)\alpha\psi]+(1-\alpha)(1-\tau_H)[(1-\phi)\eta+1]^2r^2}. \tag{35}$$

See Appendix I for the derivation of equation (35). Since the numerator of equation (35) is positive and $0<\hat{u}<1$, the denominator of equation (35) must also be positive. Then we have two possible cases: $r<(1-\tau_K)\alpha\psi$ and $r>(1-\tau_K)\alpha\psi$. Suppose $r<(1-\tau_K)\alpha\psi$ or $\rho>\Big[(1-\tau_K)\alpha-[\theta-(1-\theta)\eta]\Big]\psi$ holds. Then, the consumption tax has a positive level effect on $\hat{u}$. However, the transversality condition denoted by $\lim_{t\to\infty}\lambda_1(t)K(t)e^{-\rho t}=0$ implies that $\rho>\Big[1-[\theta-(1-\theta)\eta]\Big]\psi$[7]. Since we know that $0\leq\tau_K<1$ and $0<\alpha<1$, the transversality condition implies that $\rho>\Big[(1-\tau_K)\alpha-[\theta-(1-\theta)\eta]\Big]\psi$ also holds. Alternatively, when $r>(1-\tau_K)\alpha\psi$ or $\rho<\Big[(1-\tau_K)\alpha-[\theta-(1-\theta)\eta]\Big]\psi$ equation (35) reveals that a level effect of the consumption tax on $\hat{u}$ is ambiguous. However, the transversality condition eliminates the case, $\rho<\Big[(1-\tau_K)\alpha-[\theta-(1-\theta)\eta]\Big]\psi$.

As far as physical capital allocation in the output production sector in the steady growth equilibrium, $\hat{v}$ is concerned, we can derive it by substituting equations (34) and (35) into equation (A3) in Appendix I. Then we obtain

$$\hat{v}=\frac{1}{1+\dfrac{\beta\psi\Big[(1+\tau_C)\phi\eta[r-(1-\tau_K)\alpha\psi]+(1-\alpha)(1-\tau_H)[(1-\phi)\eta+1\Big]^2r}{\alpha(1-\tau_K)\Big[r(r-(1-\beta))[(1-\phi)\eta+1]^2+(1+\tau_C)\phi\eta\beta\psi^2\Big]}}. \tag{36}$$

It is clear from equation (36) that an increase in the consumption tax has a positive impact on $\hat{v}$.

6) $\hat{z}$ can also be denoted by

$$\hat{z}=\frac{(r-\rho)(1-\beta)}{r[\theta-(1-\theta)\eta]}.$$

Since we know taht $0<\hat{z}<1$, we have $0<(r-\rho)(1-\beta)<r[\theta-(1-\theta)\eta]$, thereby assuring that the equilibrium steady growth rate is positive.

7) After substituting $C(t)=C(0)e^{\psi t}$ and $H(t)=H(0)e^{\psi t}$ into equation (11), we insert the resultant equation and $K(t)=K(0)e^{\psi t}$ into the transversality condition, $\lim_{t\to\infty}\lambda_1(t)K(t)e^{-\rho t}=0$. Then we find that, in order to satisfy the transversality condition, $\rho>\Big[1-[\theta-(1-\theta)\eta]\Big]\psi$ must hold.

On the other hand, an increase in the capital income tax or the wage income tax has an ambiguous impact on $\hat{v}$. We denote proposition 2 to summarize the level effect of a change in the consumption tax on human capital allocation in the steady growth equilibrium.

Proposition 2

The consumption tax has no level effect on human capital allocation in the steady growth equilibrium in the education sector. However, the consumption tax has a positive level effect on human capital allocation in the steady growth equilibrium in the output production sector. Consequently, an increase in the consumption tax reduces human capital allocation toward unpaid work in the steady growth equilibrium.

Proposition 2 reveals the relation between the consumption tax policy and the education policy. It is conceivable that the government faces the policy problem such that revenue from the consumption tax has to be increased, however, somehow it must keep the current human capital allocation level in the education sector because the accumulation of human capital is considered to be one of the important growth engines.

Proposition 2 also reveals one of the outstanding differences between our model and the model without unpaid work. When we review the model with no unpaid work or when we consider a case in which $\eta=0$ and $z=1-u$, a change in the consumption tax does not affect human capital allocation in the steady growth equilibrium in the production sector and in the education sector.[8)] However, in our model, an increase in the consumption tax has a level effect on human capital allocation in the steady growth equilibrium in the output production sector, but has no level effect on that in the education sector. Consequently, agents reduce human capital allocation toward unpaid work.

5. A Centralized Economy and the Tax Policy

In this section, we examine the steady growth equilibrium in the centralized economy corresponding to that in the decentralized economy in section 3. The key question to be address is the extent to which, through appropriate tax policy, the decentralized economy can attain the planned economy (the centralized economy). To that end, we investigate the optimal tax scheme that makes the decentralized economy coincide with the centralized economy.[9)] The government as a social planner in the centralized economy maximizes equation (9) subject to the government budget constraint, the economy-wide resource constraint, initial bonds, the initial physical capital

8) If $\eta=0$ and $z=1-u$ hold, then we have $\tilde{z}=[r-(1-\beta)\tilde{\psi}]/r$ where $\tilde{\psi}=(r-\rho)/\theta$.
9) The method of searching out the optimal tax sheme basically follows Turnovsky (1995), (1996).

stock, and the initial human capital stock. The system of equations which characterize the steady growth equilibrium in the centralized economy are denoted by

$$\psi^C=\frac{r^C-\rho}{\theta-(1-\theta)\eta} \tag{37}$$

where $r^C=\left[(A\alpha)^{\beta}[(1-\beta)B]^{1-\alpha}\left[\frac{\beta(1-\alpha)}{\alpha(1-\beta)}\right]^{(1-\alpha)\beta}\right]^{\frac{1}{1-\alpha+\beta}}$;

$$\hat{z}^C=\frac{(1-\beta)\psi^C}{r^C}; \tag{38}$$

$$\hat{u}^C=\frac{(1-\alpha)\left[r^C(r^C-(1-\beta)\psi^C)[(1-\phi)\eta+1]^2+[(1-\phi)\eta+1]\phi\eta\beta\psi^{C^2}\right]}{r^C[(1-\phi)\eta+1]\phi\eta[r^C-\alpha\psi^C]+(1-\alpha)[(1-\phi)\eta+1]^2r^{C2}}; \tag{39}$$

$$\hat{v}^C=\frac{1}{1+\dfrac{\beta\psi^C\left[[(1-\phi)\eta[r^C-\alpha\psi^C]+(1-\alpha)[(1-\phi)\eta+1\right]^2r^C}{\alpha\left[r^C(r^C-(1-\beta))[(1-\phi)\eta+1]^2+[(1-\phi)\eta+1]\phi\eta\beta\psi^{C^2}\right]}}; \tag{40}$$

$$\psi^C=B\left[\frac{(1-v)K}{zH}\right]^{\beta}z. \tag{41}$$

The derivations of equations (37)〜(41) basically follow those of equations (32), (33), (34), (35), and (36). Note that we have an additional first order condition which asserts that private consumption public consumption (government expenditure) ratio is $(1-\phi)\eta$. That is why we may reasonably assume that, in the decentralized economy, agents know C/G been set $(1-\phi)\eta$. (See Appendix II.)

Before we investigate the optimal tax scheme, we assume that the social planner imposes the capital income tax and the wage income tax at the same rate or $\tau_K=\tau_H=\tau$. Then, in order to find the optimal tax scheme which leads the decentralized economy to the centralized economy, we equate equation (32) to equation (37), equation (35) to equation (39), and equation (36) to equation (40). As a result, we see that the optimal tax scheme is to impose $\tau=0$ and $\tau_C=(1-\phi)\eta$. We denote proposition 3 to summarize our findings regarding the optimal tax scheme.

Proposition 3

When the social planner sets the capital income tax rate and the wage income tax rate being the same, the optimal tax scheme is to impose no tax on both capital income and wage income and to depend all tax receipts on the consumption tax. Then, the consumption tax rate should be the private consumption—the public consumption ratio, $\tau_C=C/G=(1-\phi)\eta$.

6. Concluding Remarks

In this paper we focused on how self-sufficient goods and services involving unpaid work relate the growth of the economy and the optimal tax scheme and found three noteworthy propositions. That is, the capital income tax and the wage income tax have a negative growth effect whereas the consumption tax has no growth effect. On the other hand, an increase in the consumption tax has no level effect on human capital allocation in the education sector. Careful readers can easily realize that this result is due to the specification of self-sufficient goods and services denoted by equation (8) and the condition that the $G-C$ ratio is constant. Thus, if we assume that $L(t)=[1-u(t)-z(t)]^{\phi}G(t)^{1-\phi}$ or self-sufficient goods and services require only one unit of human capital and public consumption goods, the consumption tax also has a growth effect since the term, $u+z$ enters into the expression of the rate of return on bond holdings in the steady growth equilibrium. Alternatively, if we assume that public consumption goods is nothing but self-sufficient goods and services or $L(t)=G(t)$, then we go back to the conclusion that an increase in the consumption tax has no growth effect.

As long as empirical applications of our model is concern, to test our model is to test our specification of self-sufficient goods and services assuming that our actual economy meets the steady growth equilibrium. However, a monetary evaluation of unpaid work is fairly new attempt, and therefore, provides insufficient time series data. Further, how to measure human capital has been one of the difficult questions faced by the empirical growth literature. Some economists have employed data on school enrollment rates, public expenditures on education, and literacy rates. Barro and Lee (1993) have assembled data on average educational attainment in the population among persons 25 years old and over for a large number of countries at five year intervals going back to 1960. Various authors including Islam (1995) and Hall and Jones (1998) have regarded their data as a measure of the stock of human capital per person.

Further research can be gained by investigating transitional dynamics of our model or by introducing various specifications of self-sufficient goods and services involving unpaid work in order to examine the relationship between taxes and long-term economic growth.

Appendix I

After dividing both sides of equation (3) by K, we multiply both sides of the resultant equation by K/Y to obtain

$$[(1-\phi)\eta+1]\frac{C}{Y}=1-\frac{K}{Y}\psi. \tag{A1}$$

However, the private consumption-output ratio is derived by combining equations (11) and (12).

$$\frac{C}{Y}=\frac{(1-\tau_H)[(1-\phi)\eta+1](1-\alpha)(1-u-z)}{(1+\tau_C)\phi\eta u}. \tag{A2}$$

Further, equation (22) implies that

$$\frac{1}{v}=1+\frac{\beta(1-\alpha)(1-\tau_H)z}{\alpha(1-\beta)(1-\tau_K)u}. \tag{A3}$$

Since $Y/K=Av(vK)^{\alpha-1}(uH)^{1-\alpha}$ holds, combining equations (A1), (A2), and (A3) and remembering equations (27) and (32) yield

$$\frac{(1-\tau_H)[(1-\phi)\eta+1]^2(1-\alpha)(1-u-z)}{(1+\tau_C)\phi\eta u}=1-\frac{(1-\tau_K)\alpha}{\rho+[\theta-(1-\theta)\eta]\psi}\left[1+\frac{\beta(1-\alpha)(1-\tau_H)z}{\alpha(1-\beta)(1-\tau_K)u}\right]\psi \qquad (A4)$$

where z is denoted by equation (34). Then we can express equation (A4) in terms of u to obtain equation (35).

Appendix II

The social planner in a centralized economy chooses paths of C, K, H, H, S, u, v, and z to maximize equation (1) subject to equation (2), the initial physical capital stock, the initial human capital stock, initial bond holdings, and the budget constraint denoted by

$$\dot{K}(t)+\dot{S}(t)=A[v(t)K(t)]^{\alpha}[u(t)H(t)]^{1-\alpha}+r(t)S(t)-C(t)-G(t)$$

taking a path of $r(t)$ as given. Therefore, we set up the current value Hamiltonian in the centralized economy,

$$J^C(C,K,H,S,u,v,z,\lambda_1^C,\lambda_2^C,t)=\frac{\left[C[(1-u-z)H]^{\phi\eta}G^{(1-\phi)\eta}\right]^{1-\theta}-1}{1-\theta}$$
$$+\lambda_1^C\left[A[vK]^{\alpha}[uH]^{1-\alpha}+r(t)S-C-G\right]$$
$$+\lambda_2^C\left[B[(1-v)K]^{\beta}[zH]^{1-\beta}\right].$$

where λ_1^C and λ_2^C are the shadow value of physical capital and that of human capital in the centralized economy, respectively.

The necessary conditions are

$$\frac{\partial J^C}{\partial C}=0;\ [(1-u-z)H]^{\phi\eta}G^{(1-\phi)\eta}[D^C]^{-\theta}-\lambda_1^C=0: \qquad (A\,II\text{-}1)$$

$$\frac{\partial J^C}{\partial u}=0;\ -\phi\eta C[(1-u-z)H]^{\phi\eta-1}G^{(1-\phi)\eta}[D^C]^{-\theta}+(1-\alpha)A\left[\frac{vK}{uH}\right]^{\alpha}\lambda_1^C=0: \qquad (A\,II\text{-}2)$$

$$\frac{\partial J^C}{\partial z}=0;\ -\phi\eta C[(1-u-z)H]^{\phi\eta-1}G^{(1-\phi)\eta}[D^C]^{-\theta} \qquad (A\,II\text{-}3)$$
$$+(1-\beta)B\left[\frac{(1-v)K}{zH}\right]^{\beta}\lambda_2^C=0:$$

$$\frac{\partial J^C}{\partial v}=0;\ \alpha A\left[\frac{vK}{uH}\right]^{\alpha-1}\lambda_1^C-B\beta\left[\frac{(1-v)K}{zH}\right]^{\beta-1}\lambda_2^C=0: \qquad (A\,II\text{-}4)$$

$$\frac{\partial J^C}{\partial G}=0;\ (1-\phi)\eta C[(1-u-z)H]^{\phi\eta}G^{(1-\phi)\eta-1}[D^C]^{-\theta}-\lambda_1^C=0: \qquad (A\,II\text{-}5)$$

$$-\dot{\lambda}_1^C+\lambda_1^C\rho=\frac{\partial J^C}{\partial K};\ -\dot{\lambda}_1^C+\lambda_1^C\rho=A\alpha v\left[\frac{vK}{uH}\right]^{\alpha-1}\lambda_1^C+B\beta(1-v)\left[\frac{(1-v)K}{zH}\right]^{\beta-1}\lambda_2^C: \qquad (A\,II\text{-}6)$$

$$-\dot{\lambda}_1^C+\lambda_1^C\rho=\frac{\partial J^C}{\partial S};\qquad -\dot{\lambda}_1^C+\lambda_1^C\rho=r\lambda_1^C: \qquad (A\,II\text{-}7)$$

$$-\dot{\lambda}_2^C+\lambda_2^C\rho=\frac{\partial J^C}{\partial H};$$

$$-\dot{\lambda}_2^C+\lambda_2^C\rho=\phi\eta(1-u-z)C[(1-u-z)H]^{\phi\eta-1}G^{(1-\phi)\eta}[D^C]^{-\theta} \tag{A II-8}$$
$$+A(1-\alpha)u\left[\frac{vK}{uH}\right]^{\alpha-1}\lambda_1^C+(1-\beta)Bz\left[\frac{(1-v)K}{zH}\right]^{\beta}\lambda_2^C.$$

where $D^C=C[(1-u-z)H]^{\phi\eta}G^{(1-\phi)\eta}$ and the transversality conditions,

$$\lim_{t\to\infty}\lambda_1^C(t)K(t)e^{-\rho t}=0;$$
$$\lim_{t\to\infty}\lambda_1^C(t)S(t)e^{-\rho t}=0;$$
$$\lim_{t\to\infty}\lambda_2^C(t)H(t)e^{-\rho t}=0.$$

The equilibrium steady growth rate and physical capital allocation and human capital allocation in the steady growth equilibrium can be derived by following the same fashion in the decentralized economy case. Equation (A II-5) is an additional necessary condition. Equations (A II-1) and (A II-5) imply that the social planner sets $G/C=(1-\phi)\eta$. Thus it is admissible to assume that $G/C=(1-\phi)\eta$ is the common knowledge for the representative agent in the decentralized economy.

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